Answer in your binder

8. How can you find a unit rate when given a rate?

_____________________________________________________________________

_____________________________________________________________________

_____________________________________________________________________

_____________________________________________________________________

EXPLORING ACTIVITY

**Discovering Proportional Relationships**

Many real-world situations can be described by proportional relationships. Proportional relationships have special characteristics.

A giant tortoise moves at a slow but steady pace. It takes the giant tortoise 3 seconds to travel 10.5 inches.

A. Use the bar diagram to help you determine how many inches a tortoise travels in 1 second. What operation did you use to find the answer?

B. Complete the table.

<table>
<thead>
<tr>
<th>Time (sec)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (in.)</td>
<td></td>
<td>10.5</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

C. For each column of the table, find the distance and the time. Write each fraction as a decimal. Put distance in the numerator and time in the denominator.

\[
\text{Distance} / \text{Time} = \text{Decimal Value}
\]

D. What do you notice about the decimal forms of the fractions?

E. **Conjecture** How do you think the distance a tortoise travels is related to the time?
EXPLORE ACTIVITY (cont’d)

Reflect

1. Suppose the tortoise travels for 12 seconds. Explain how you could find the distance the tortoise travels.

2. How would you describe the rate of speed at which a tortoise travels?
Proportional Relationships

A rate of change is a rate that describes how one quantity changes in relation to another quantity. A proportional relationship between two quantities is one in which the rate of change is constant, or one in which the ratio of one quantity to the other is constant.

Any two rates or ratios based on a given proportional relationship can be used to form a proportion. A proportion is a statement that two rates or ratios are equivalent, for example, \( \frac{6 \text{ mi}}{2 \text{ h}} = \frac{3 \text{ mi}}{1 \text{ h}} \), or \( \frac{\frac{2}{4}}{\frac{1}{2}} = \frac{1}{1} \).

**EXAMPLE 1**

Callie earns money by dog sitting. Based on the table, is the relationship between the amount Callie earns and the number of days a proportional relationship?

<table>
<thead>
<tr>
<th>Number of Days</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amount Earned ($)</td>
<td>16</td>
<td>32</td>
<td>48</td>
<td>64</td>
<td>80</td>
</tr>
</tbody>
</table>

**STEP 1**

Write the rates.

\[
\frac{\text{Amount earned}}{\text{Number of days}} = \frac{\$16}{1 \text{ day}}
\]

\[
\frac{\$16}{2 \text{ days}} = \frac{\$32}{1 \text{ day}}
\]

\[
\frac{\$16}{3 \text{ days}} = \frac{\$48}{1 \text{ day}}
\]

\[
\frac{\$16}{4 \text{ days}} = \frac{\$64}{1 \text{ day}}
\]

\[
\frac{\$16}{5 \text{ days}} = \frac{\$80}{1 \text{ day}}
\]

Put the amount earned in the numerator and the number of days in the denominator.

Each rate is equal to \( \frac{\$16}{1 \text{ day}} \) or \( \$16 \) per day.

**STEP 2**

Compare the rates. The rates are all equal. This means the rate is constant, so the relationship is proportional.

The constant rate of change is \( \$16 \) per day.
3. The table shows the distance Allison drove on one day of her vacation. Is the relationship between the distance and the time a proportional relationship? Did she drive at a constant speed? Explain.

<table>
<thead>
<tr>
<th>Time (h)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (mi)</td>
<td>65</td>
<td>120</td>
<td>195</td>
<td>220</td>
<td>300</td>
</tr>
</tbody>
</table>
Writing an Equation for a Proportional Relationship

If there is a proportional relationship between \( x \) and \( y \), you can describe that relationship using the equation \( y = kx \). The variable \( k \) is called the **constant of proportionality**, and it represents the constant rate of change or constant ratio between \( x \) and \( y \). The value of \( k \) is represented by the equation \( k = \frac{y}{x} \).

**EXAMPLE 2**

Two pounds of cashews shown cost $19, and 8 pounds cost $76. Show that the relationship between the number of pounds of cashews and the cost is a proportional relationship. Then write an equation for the relationship.

**STEP 1**

Make a table relating cost in dollars to pounds.

<table>
<thead>
<tr>
<th>Number of Pounds</th>
<th>2</th>
<th>3</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost ($)</td>
<td>19</td>
<td>28.50</td>
<td>76</td>
</tr>
</tbody>
</table>

**STEP 2**

Write the rates. Put cost in the numerator and pounds in the denominator. Then simplify each rate.

\[
\frac{\text{Cost}}{\text{Number of Pounds}} = \frac{19}{2} = 9.50 \quad \frac{28.50}{3} = 9.50 \quad \frac{76}{8} = 9.50
\]

The rates are all equal to $9.50 per pound. They are constant, so the relationship is proportional. The constant rate of change is $9.50 per pound.

**STEP 3**

To write an equation, first tell what the variables represent.

- Let \( x \) represent the number of pounds of cashews.
- Let \( y \) represent the cost in dollars.
- Use the numerical part of the constant rate of change as the constant of proportionality.

So, the equation for the relationship is \( y = 9.5x \).
4. For a school field trip, there must be 1 adult to accompany 12 students, 3 adults to accompany 36 students, and 5 adults to accompany 60 students. Show that the relationship between the number of adults and the number of students is a proportional relationship. Then write an equation for the relationship.

<table>
<thead>
<tr>
<th>Number of students</th>
<th>12</th>
<th>36</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of adults</td>
<td>1</td>
<td>3</td>
<td>5</td>
</tr>
</tbody>
</table>

_______________________________________________

_______________________________________________
1. Based on the information in the table, is the relationship between time and the number of words typed a proportional relationship? (Explore Activity and Example 1)

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of words</td>
<td>45</td>
<td>90</td>
<td>135</td>
<td>180</td>
</tr>
</tbody>
</table>

\[
\frac{\text{Number of words}}{\text{Minutes}} = \frac{45}{1} = \frac{90}{2} = \frac{135}{3} = \frac{180}{4} = \square = \square = \square = \square
\]

The relationship **is** proportional.

Find the constant of proportionality \( k \). Then write an equation for the relationship between \( x \) and \( y \). (Example 2)

2. | \( x \) | 2 | 4 | 6 | 8 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
</tr>
</tbody>
</table>

3. | \( x \) | 8 | 16 | 24 | 32 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
</tr>
</tbody>
</table>
4. How can you represent a proportional relationship using an equation?